Practice with Expected Value and Fair Games

1. You draw one card from a standard deck of playing cards. If you pick a heart, you will win $10. If you pick a face card, which is not a heart, you win $8. If you pick any other card, you lose $6. Do you want to play? Explain.

\[
\text{expected value} = \left(\frac{13}{52}\right)(10) + \left(\frac{9}{52}\right)(8) + \left(\frac{30}{52}\right)(-6) = .42
\]

expected value is you win $.42, so you want to play

2. The world famous gambler from Philadelphia, Señor Rick, proposes the following game of chance. You roll a fair die. If you roll a 1, then Señor Rick pays you $25. If you roll a 2, Señor Rick pays you $5. If you roll a 3, you win nothing. If you roll a 4 or a 5, you must pay Señor Rick $10, and if you roll a 6, you must pay Señor Rick $15. Is Señor Rick crazy for proposing such a game? Explain.

\[
\text{expected value} = \left(\frac{1}{6}\right)(25) + \left(\frac{1}{6}\right)(5) + \left(\frac{1}{6}\right)(0) + \left(\frac{2}{6}\right)(-10) + \left(\frac{1}{6}\right)(-15) = -.83
\]

Señor Rick is not crazy since the expected value is you lose $ .83.

3. You pay $10 to play the following game of chance. There is a bag containing 12 balls, five are red, three are green and the rest are yellow. You are to draw one ball from the bag. You will win $14 if you draw a red ball and you will win $12 if you draw a yellow ball. How much do you expect to win or loss if you play this game 100 times?

\[
\text{expected value of 1 game} = \left(\frac{5}{12}\right)(4) + \left(\frac{4}{12}\right)(2) + \left(\frac{3}{12}\right)(-1) = -.1667
\]

\[
\text{expected value of 100 games} = 100(-.1667) = -$16.67, you loose $16.67
\]

4. A detective figures that he has a one in nine chance of recovering stolen property. His out-of-pocket expenses for the investigation are $9,000. If he is paid his fee only if he recovers the stolen property, what should he charge clients in order to breakeven?

want an expected value of 0, let X = what he should charge

Solve: \(\left(\frac{1}{9}\right)(X - 9000) + \left(\frac{8}{9}\right)(-9000) = 0\)

\(X = 81000, \text{so the detective must charge }$81,000.

5. At Tucson Raceway Park, your horse, Soon-to-Be-Glue, has a probability of 1/20 of coming in first place, a probability of 1/10 of coming in second place, and a probability of ¼ of coming in third place. First place pays $4,500 to the winner, second place $3,500 and third place $1,500. Is it worthwhile to enter the race if it costs $1,000?

\[
\text{expected value} = \left(\frac{1}{20}\right)(3500) + \left(\frac{1}{10}\right)(2500) + \left(\frac{1}{4}\right)(1250) + \left(\frac{3}{5}\right)(-100) = -$50
\]

Not worthwhile to enter the race, he is expected to loose $50.

6. Your company plans to invest in a particular project. There is a 35% chance that you will lose $30,000, a 40% chance that you will break even, and a 25% chance that you will make $55,000. Based solely on this information, what should you do?

\[
\text{expected value} = .35(-30000) + .40(0) + .25(55000) = 3250
\]

You expect to make $3,250, so proceed with the project.

7. A manufacturer is considering the manufacture of a new and better mousetrap. She estimates the probability that the new mousetrap is successful is 0.75. If it is successful it would generate profits of $120,000. The development costs for the mousetrap are $98,000. Should the manufacturer proceed with plans for the new mousetrap? Why or why not?

\[
\text{expected value} = .75(120000) + .25(-98000) = 65,500
\]

Expected profit of 65,500, proceed with the project.
8. A grab bag contains 12 packages worth 80 cents apiece, 15 packages worth 40 cents each and 25 packages worth 30 cents apiece. Is it worthwhile to pay 50 cents for the privilege of picking one of the packages at random?

\[
\text{expected value of your prize} = \left(\frac{12}{52}\right)(.80) + \left(\frac{15}{52}\right)(.40) + \left(\frac{25}{52}\right)(.30) = .44
\]

Expected value of your prize is $0.44, if you paid $0.50 then you lost money.

9. You pay $3.00 to play. The dealer deals you one card. If it is a spade, you get $10. If it is anything else, you lose your money. Is this game fair?

\[
\text{expected value for you} = \left(\frac{13}{52}\right)(7) + \left(\frac{39}{52}\right)(-3) = -.50
\]

You expect to lose $.50 so not a fair game.

10. A casino game costs $3.50 to play. You draw 1 card from a deck. If it is a heart, you win $10; If it is the Queen of hearts, you win $50. Is this a fair game?

\[
\text{expected value for you} = \left(\frac{12}{52}\right)(6.5) + \left(\frac{1}{52}\right)(46.5) + \left(\frac{39}{52}\right)(-3.5) = -.23
\]

You expect to lose $.23 so not a fair game.

11. A player rolls a die and receives the number of dollars equal to the number on the die EXCEPT when the die shows a 6. If a 6 is rolled, the player loses $6. If the game is to be fair, what should be the cost to play?

\[
\text{expected value} = 1(\frac{1}{6}) + 2(\frac{1}{6}) + 3(\frac{1}{6}) + 4(\frac{1}{6}) + 5 \left(\frac{1}{6}\right) - 6(\frac{1}{6}) = 1.5
\]

The expected value is $1.5 when games costs nothing so charge $1.50 to make the game fair (expected value = 0).

\[-.5(\frac{1}{6}) + .5(\frac{1}{6}) + 1.5(\frac{1}{6}) + 2.5(\frac{1}{6}) + 3.5(\frac{1}{6}) - 7.5(\frac{1}{6}) = 0\]

12. Consider the above game with a modification. We would like to make a fair, FREE game. We will do this by charging a customer money if they roll a 1. If all the rest is the same, what should we charge if they roll a 1?

\[
\text{fair game, so expected value} = 0 , X = \text{outcome for a 1}
\]

\[0 = X(\frac{1}{6}) + 2(\frac{1}{6}) + 3(\frac{1}{6}) + 4(\frac{1}{6}) + 5 \left(\frac{1}{6}\right) - 6(\frac{1}{6})\]

Solving for X we get = $-8;
We should charge $8 if they roll a 1.

13. This last game costs $1 to play. You are given a coin to flip. Any time you flip tails, the game ends. If you flip heads, you may flip again for a max of 5 flips. You will be paid $1 for each head. If all 5 flips result in heads, you win the $5 for 5 heads plus a $2 bonus. Is this a fair game?

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\[
\text{expected value} = -1(\frac{1}{2}) + 0(\frac{1}{4}) + 1(\frac{1}{8}) + 2(\frac{1}{16}) + 3(\frac{1}{32}) + 6(\frac{1}{32}) = .03
\]

not a fair game, you expect to win $.03 per game